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PROBLEM OF MEASURING THE THERMOPHYSICAL CHARACTERISTICS OF THIN RESISTIVE AND DIELECTRIC FILMS

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We consider the theoretical and experimental possibilities of determining a complex of thermophysical characteristics of thin films on a substrate, on the basis of a nonstationary method with boundary conditions of the fourth kind.

In order to calculate the thermal regimes of operation of microcircuits and thermoprinting matrices, and in a number of other cases, it is necessary to know the thermophysical characteristics of individual film components on a substrate. Such investigations are of interest for their own sake as well [1, 2]. In recent years the greatest amount of attention is being devoted to complex investigation methods, when a single experiment serves to determine all the independent thermophysical characteristics [2, 3]. Pugachev, Volkov, and Churakova [2] gave a detailed analysis of the methods applicable to thin (10-10^2 nm) films of metals without substrates. The investigation of films on substrates is discussed in [4-10], and their authors use mainly pulsed heat sources. The present study is based on the solution of a nonstationary heat-conduction equation with boundary conditions of the fourth kind. A system consisting of a semibounded substrate (A) and a film (F) with an initial temperature of T_{00} (Fig. 1) is brought into thermal contact at time $\tau = 0$ with a semibounded body (B) which has an initial temperature of T_{10}. The heat-conduction equation for this case has the form

$$\frac{\partial T_1(x,\ \tau)}{\partial \tau} = a_1 \frac{\partial^2 T_1(x,\ \tau)}{\partial x^2} \quad (\tau > 0,\ x > 0),$$
$$\frac{\partial T_0(x,\ \tau)}{\partial \tau} = a_0 \frac{\partial^2 T_0(x,\ \tau)}{\partial x^2} \quad (\tau > 0,\ 0 > x > -x_0), \tag{1}$$
$$\frac{\partial T_2(x,\ \tau)}{\partial \tau} = a_2 \frac{\partial^2 T_2(x,\ \tau)}{\partial x^2} \quad (\tau > 0,\ x < -x_0).$$

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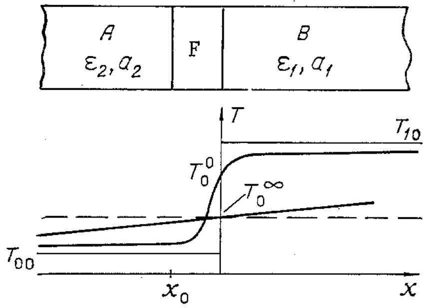

Fig. 1

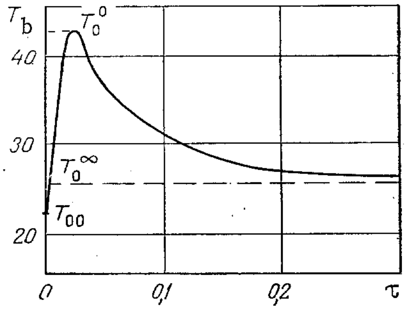

Fig. 2

Fig. 1. Temperature field of the system consisting of semibounded substrate and film and a semibounded body.

Fig. 2. Temperature at the interface as a function of time ($\varepsilon_0 < \varepsilon_2$). τ, sec; T, °C.

The origin lies in the plane of contact. We write the boundary conditions:

$$T_1(0,\ \tau) = T_0(0,\ \tau),\quad T_0(-x_0,\ \tau) = T_2(-x_0,\ \tau),$$
$$\frac{\partial T_1(0,\ \tau)}{\partial x} = \frac{\lambda_0}{\lambda_1}\ \frac{\partial T_0(0,\ \tau)}{\partial x},\quad \frac{\partial T_0(-x_0,\ \tau)}{\partial x} = \frac{\lambda_2}{\lambda_0}\ \frac{\partial T_2(-x_0,\ \tau)}{\partial x}, \tag{2}$$
$$T_1(x,\ 0) = T_{10},\quad T_0(x,\ 0) = T_2(x,\ 0) = T_{00},$$
$$\frac{\partial T_1(+\infty,\ \tau)}{\partial x} = \frac{\partial T_2(-\infty,\ \tau)}{\partial x} = 0,$$

where T_1, T_0, T_2 are the temperatures of the body B, the film, and the body A (the substrate), respectively; a_1, a_0, a_2, thermal diffusivities; λ_1, λ_0, λ_2, thermal conductivities of these bodies; x_0, thickness of the film F.

This equation can be solved by the Laplace transform method. The solution for the film at the boundary x = 0 is:

$$T_0(0,\ \tau) = T_{00} + \Delta T\left[\frac{1}{K_1+1}\sum_{n=0}^{\infty} h^n \operatorname{erfc}\frac{nx_0}{\sqrt{a_0\tau}} + \frac{K_2-1}{1+K_1+K_2+K_1K_2}\sum_{n=0}^{\infty} h^n \operatorname{erfc}\frac{(n+1)\,x_0}{\sqrt{a_0\tau}}\right], \tag{3}$$

where

$$\Delta T = T_{10} - T_{00};\quad K_1 = \frac{\varepsilon_0}{\varepsilon_1};\quad K_2 = \frac{\varepsilon_0}{\varepsilon_2};$$
$$h = \frac{1-K_1-K_2+K_1K_2}{1+K_1+K_2+K_1K_2};$$

$\varepsilon = \lambda/\sqrt{a}$ is the thermal activity. As $\tau \to 0$, we obtain from (3) the equation

$$T_0^0 = T_0(0,\ \tau \to 0) = T_{00} + \Delta T\ \frac{\varepsilon_1}{\varepsilon_1+\varepsilon_0}, \tag{4}$$

i.e., the solution of the problem for two bodies having thermal activity values ε_1 and ε_0 brought into contact [11, 12].

Thus, the initial value of the temperature on the boundary after contact enables us to determine the value of ε_0 for known values of T_{10}, T_{00}, ε_1. As $\tau \to \infty$, Eq. (3) becomes

$$T_0^\infty = T_0(0,\ \tau \to \infty) = T_{00} + \Delta T\ \frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2}, \tag{5}$$

which is the solution of the problem of two bodies having thermal activity values of ε_1 and ε_2. As can be seen from Eq. (3), the time-dependent variation of the temperature on the boundary is determined by the thermal diffusivity of the film under investigation, a_0, and also by the coefficients K_1, K_2, h, which depend on the relation between the thermal-activity values of the media described. If the thermal activity of the film is determined from Eq.

(4), these coefficients take on certain numerical values, and Eq. (3) can serve to determine the thermal diffusivity of the film being investigated. Analogous results are obtained if we take the solution of the equation for the body B on the boundary x = 0:

$$T_1(x,\ \tau) = T_{10} + \Delta T\left[\frac{K_1(K_2 - 1)}{1 + K_1 + K_2 + K_1K_2}\sum_{n=0}^{\infty} h^n \operatorname{erfc}\left(\frac{(n+1)x_0}{\sqrt{a_0\tau}} + \frac{x}{2\sqrt{a_0\tau}}\right) + \frac{K_1}{K_1+1}\sum_{n=0}^{\infty} h^n \operatorname{erfc}\left(\frac{nx_0}{\sqrt{a_0\tau}} + \frac{x}{2\sqrt{a_0\tau}}\right)\right]. \quad (6)$$

For values of $|h| < 0.5$ the series (3), (6) are rapidly convergent; however, it is not efficient to represent the solution in analytic form, and it is more convenient to carry out the determination of the thermal diffusivity by the method of successive approximations using a computer. For multiple measurements at constant values of ε_0, ε_1, ε_2, x_0 it is possible to construct a family of curves $T_0(\tau)$ for different values of a_0.

In the present study we investigated resistive films 20-25 μm thick, obtained by the method of stencil printing on silicon substrates ($\varepsilon_2 = 2680$ J/m²·deg·sec$^{1/2}$). As the body B we used distilled water with $\varepsilon_1 = 1600$ J/m²·deg·sec$^{1/2}$, which was in a thermostat at a temperature of 42°C.

The temperature on the boundary was recorded by means of low-inertia thermal resistances [5] 0.1-0.2 μm thick, obtained by the method of vacuum atomization onto the surface of the film being investigated. The thermal resistance was included in an arm of a bridge whose imbalance signal was amplified and transmitted to a recording oscillograph. The typical form of the time-dependent variation of $T_0(0,\ \tau)$ for $\varepsilon_0 < \varepsilon_2$ is shown in Fig. 2. The maximum of this curve in the case shown determines the value of the temperature T_0^0. For $\varepsilon_0 > \varepsilon_2$ there should be a break-point at this part of the curve, after which the temperature will increase smoothly to the value T_0^∞.

In [11] it was noted that in the two-body problem we may disregard the effect of the lateral heat exchange and the finiteness of the bodies for $\tau = 0.01$-1.5 sec. In our experiment, for $\tau = 0.02$-0.2 sec, in the film under investigation, we found a thermal-activity value of $\varepsilon_0 = 1370$ J/m²·deg·sec$^{1/2}$ and a thermal diffusivity of $a_0 = 8\cdot10^{-9}$ m²/sec. The sensitivity of the method to thermal diffusivity is determined by the curvature of the time-dependent graph of the temperature on the boundary, described by Eq. (3), or, in other words, by the difference between the temperatures T_0^0 and T_0^∞. Starting from Eqs. (4) and (5), we can conclude that the maximum sensitivity to thermal diffusivity will be found in the case when the thermal-activity values of the film and the substrate are farthest apart. The value of the thermal conductivity is calculated from the formula $\varepsilon = \lambda/\sqrt{a}$.

NOTATION

τ, time; x, coordinate perpendicular to the film surface; ε_2, ε_0, ε_1, coefficients of thermal activity ($\varepsilon = \lambda/\sqrt{a}$) of the substrate, the film, and the second body; T_{00}, initial temperature of the film–substrate system; T_{10}, initial temperature of the second body; $\Delta T = T_{10} - T_{00}$; T_0^0, temperature at the boundary after contact as $\tau \to 0$; T_0^∞, temperature at the boundary after contact as $\tau \to \infty$; $K_1 = \varepsilon_0/\varepsilon_1$; $K_2 = \varepsilon_0/\varepsilon_2$; $h = (1 - K_1 - K_2 + K_1K_2/1 + K_1 + K_2 + K_1K_2)$.

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APPLICATION OF THE VARIATIONAL PRINCIPLE TO THE SOLUTION OF GENERALIZED COUPLED PROBLEMS IN THERMOELASTICITY OF INHOMOGENEOUS MEDIA

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A variational principle is formulated for coupled thermoelasticity for inhomogeneous media. The problem of thermoelastic energy dissipation accompanying transverse oscillations of an inhomogeneous isotropic cantilevered beam is solved.

The application of direct methods to the solution of coupled problems in thermoelasticity for inhomogeneous media encounters considerable mathematical difficulties. The development of approximate methods for solving coupled problems based on variational principles is promising.

We shall formulate the variational principle for coupled thermoelasticity for inhomogeneous media. We shall examine the isothermal energy of deformation

$$W=\frac{1}{2}\int\limits_{\Omega} c_{ijkl}(x_s)\, e_{kl}e_{ij}dV, \tag{1}$$

where Ω is the volume of the body.

Let us transform (1) taking into account the Duhamel—Neumann equation for inhomogeneous media and the equations of motion. As a result, we obtain

$$\int\limits_{\Omega} X_i\delta u_i dV+\int\limits_{A} P_i\delta u_i dA-\int\limits_{\Omega}\rho(x_s)\,\ddot{u}_i\delta u_i dV=\delta W-\int\limits_{\Omega}\beta_{ij}(x_s)\,t\delta e_{ij}dV, \tag{2}$$

where A is the surface of the body.

We introduce the vector $\vec{H}$, related to the heat flux vector by the relation

$$\vec{q}=t_0\dot{\vec{H}}. \tag{3}$$

Taking into account the generalized law of heat conduction

$$lq_i=-\lambda_{ij}^{t}(x_s)\,t_{,j} \tag{4}$$

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